On determinations of the pion3He3H coupling constant

Klieb, L; Rood, HPC

Published in:
Physics Letters B

DOI:
10.1016/0370-2693(82)90960-1

ON DETERMINATIONS OF THE PION–^{3}He–^{3}H COUPLING CONSTANT

L KLIEB and H P C ROOD
Institute for Theoretical Physics, University of Groningen, 9700 A V Groningen, The Netherlands

Received 4 February 1982
Revised manuscript received 22 February 1982

Various determinations in the current literature of the π–^{3}He–^{3}H coupling constant are reexamined, and the ones leading to very low values are shown either to be erroneous or not decisive and implausible A dispersion relation and the impulse approximation which use the same kind of approximations both lead to $G_{\pi-^3\mathrm{He}-^3\mathrm{H}}(-m_\pi^2) \approx -1$ 7, in absolute value higher than the elementary nucleon value $g_{\pi pn}(-m_\pi^2) = 1$ 41

1 Introduction In the last decade a large number of attempts has been made to determine the π–^{3}He–^{3}H coupling constant G_π [‡1] which is the pion–^{3}He–^{3}H form factor with the pion on mass shell in the elementary-particle model (EPM) The published numbers fall roughly into two ranges On the one hand several authors [1–5] have determined a value for G_π much lower than the elementary pion–nucleon coupling constant [$g_\pi \equiv g_{\pi pn}(-m_\pi^2) = 1$ 41], i e

$$G_\pi \approx \ 1\ 0 \tag{1}$$

On the other hand there are determinations, e g refs [6–9] which lead to values higher in absolute value than g_π

$$-1\ 8 \lesssim G_\pi \lesssim -1\ 5 \tag{2}$$

There seems to be a serious and not understood discrepancy Moreover various different methods are involved in both the high and low determinations which seem to be unrelated This letter aims to point out the connection between the different methods and to offer a critical discussion We hold that all determinations leading to low values contain either serious errors or underestimate the uncertainties so much that a high value cannot be excluded We will also give a calculation in the impulse approximation (IA) leading to a high value and compare with dispersion relations The discussion will be kept brief and more complete results are planned to be given elsewhere [10]

2 Impulse approximation Clearly a pion–nucleus coupling constant is a quantity defined within the EPM However, the formalism of Delorme [11] gives, in the Breit frame ($q_0 = 0$), a connection between the EPM and the IA Then

$$G_\pi(q^2) = 6^{-1/2} g_\pi(q^2)\{[\boldsymbol{\sigma}]^{0,1} + [\boldsymbol{\sigma}]^{2,1}\}, \tag{3}$$

where $[\boldsymbol{\sigma}]^{\alpha,1}$, $\alpha = 0, 2$, are reduced matrix elements defined in ref [11] A realistic wavefunction is used calculated from the RSC potential working in the $^1S_0$ and $^3S_1$–$^3D_1$ channels [12,13] We make the extrapolation to the pion pole ($q_0 = 0$, $|\boldsymbol{q}| = \mathrm{i}m_\pi$) directly (cf eq (48) of ref [14]) in the functions $j_l(qr)$, $l = 0$, 2, occurring in the radial integrals, and obtain $G_\pi = -1$ 68 Thus it is seen that the influence of the nuclear form factor leads in a natural way to values for $|G_\pi/g_\pi| > 1$ This forms a reference point for corrections we argue that any possible low value has to be compared not with g_π but with the IA value $G_\pi \approx -1$ 7

3 Dispersion relations Another way to determine $G_\pi(-m_\pi^2)$ is to use a dispersion relation From the divergence of the axial current follows

[‡1] By definition $G_\pi \equiv G_{\pi-^3\mathrm{He}-^3\mathrm{H}}(-m_\pi^2) = \sqrt{2}\, G_{\pi-^3\mathrm{He}-^3\mathrm{He}}(-m_\pi^2)$ In refs [1–5] values for $f^2 = [G_{\pi-^3\mathrm{He}-^3\mathrm{He}}(-m_\pi^2)]^2/4\pi$ are given

$$D(q^2) = -G_A(q^2) + \frac{q^2}{2Mm_\mu} G_P(q^2)$$
$$= \frac{a_\pi m_\pi^2 G_\pi}{q^2 + m_\pi^2} + I(q^2) = \frac{a_\pi m_\pi^2 G_\pi(q^2)}{q^2 + m_\pi^2}, \tag{4}$$

with $G_\pi(q^2)$ the π ^{3}He–^{3}H form factor, $a_\pi = 0.94$, $I(q^2)$ the non-π-pole contribution to the divergence and M the $A = 3$ mass. A dispersion relation by Jarlskog and Yndurain [6] fixes $I(q^2)$ under the assumption that only the anomalous cut starting at $q^2_{\text{th}} \approx -(1.8\, m_\pi)^2$ resulting from ^{3}He $\to$ dp break up contributes. From their work $I(0) = 0.21$ follows. The ^{3}H β-decay rate gives $G_A(0) = 1.22$ [8]. Not noticed in ref. [6], this leads with eq. (4) for $q^2 = 0$ (which is the Goldberger–Treiman relation in the nuclear case) necessarily to $G_\pi = -1.52$.

We repeated the dispersion relation calculations from refs. [6,7] with more recent experimental data on coupling constants [15]. Following ref. [7] the cuts from the anomalous thresholds from ^{3}He $\to$ dp and ^{3}He $\to$ d*(singlet)p break up are included, for the d* ($J = 0, T = 1$) by assuming that all strength is located in a resonance just above threshold [7]. We find $G_\pi \approx -1.7$, the exact value being somewhat dependent on the chosen integration limit in the dispersion integral.

This value is fully consistent with the value found in section 2 in the IA. As the used diagrams correspond exactly to the impulse approximation without exchange, this agreement is *not accidental*.

4. Charge exchange p^{3}He $\to$ n^{3}H, refs. [1,2,16]. We turn now to a discussion of the various determinations in the current literature seen in the light of the above calculations.

Experimental results for the charge-exchange reaction p^{3}He $\to$ n^{3}H were obtained by Bizard et al. [16]. The reaction mechanism is supposed to be mainly π-exchange.

The data have been analyzed in this reference by extrapolating the ratio $(\mathrm{d}\sigma/\mathrm{d}t)$(p^{3}He $\to$ n^{3}H)/$(\mathrm{d}\sigma/\mathrm{d}t)$(np $\to$ pn) to the pion pole. This ratio gives G_π/g_π. Using 6 data points at 415 MeV per nucleon and a straight line for the fit, the there obtained result is $G_\pi = -1.35 \pm 0.08$, i.e. slightly lower in absolute value than g_π. We estimate $\chi^2 \approx 5$. However, as already also suggested in this reference, the used ratio is, in the IA, expected to vary as a nuclear form factor. If this form factor is taken as

$$\frac{1}{3}\left|\int \boldsymbol{\sigma}_i\right|^2 = \tfrac{1}{3}\left|\langle\psi_f|\sum_{j=1}^{A} \exp[\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r}(j)]\,\boldsymbol{\sigma}(j)\,\tau_-(j)|\psi_i\rangle\right|^2,$$

as it is for the pion contribution, we can fit the data also with about the same χ^2, but with no free parameters. Here the same wavefunction as in section 2 is used. This would imply $G_\pi \approx -1.7$ like found in section 2. Both determinations do not say *too* much, as the nucleon data have large error bars, larger than the $A = 3$ ones, but they suggest that the found experimental cross sections are consistent with the impulse approximation, and so with a high G_π.

The same data have also been analyzed in refs. [1] and [2]. In ref. [1] a mapping method has been used to extrapolate data on the p^{3}H $\to$ n^{3}He differential cross section to the pion pole, and a low value [eq. (1)] was obtained. As also stated in ref. [2] mapping methods underestimate in general the uncertainties in the extrapolation much [17]. Therefore in ref. [2] an explicit parametrization has been used for the continuation, leading also to about the same low value for G_π. We would like to make the following brief remarks on this work without being able to settle all difficult points.

In the explicit parametrization no allowance is made for the fact that from the γ_5-coupling, one-pion contributions are proportional to t. From the relation $t/(t - m_\pi^2) = 1 + m_\pi^2/(t - m_\pi^2)$ this means introduction of a constant background. The same goes for the anomalous threshold contribution because it can be described rather well by a pole in the considered region. Thus there is a high background, instead of one compatible with zero, as stated in ref. [2]. However, this constant background cannot be the almost constant u-channel deuteron exchange pole: we checked that with values of the He–d–p and H–d–p couplings taken from refs. [6,15] the contribution of the deuteron pole is negligible compared with the pion pole (except for t very close to zero). It is unclear what else this background could be.

So far these remarks on the *interpretation* of the formalism of ref. [2]. The most serious objection, however, is the following. The used parametrization does not take into account the very complicated (and theoretically not fully understood) neutron–proton scattering [18]. This process cannot be described by a simple γ_5-coupling with pion propagator, as the differential cross section has no dip at $t = 0$. From the IA we know then that also the EPM parametrization must have the same complexity

In this context it may be noted that also the so much related processes of pion photoproduction (section 7) and radiative pion capture [19] are well described by the IA

The data can therefore not be expected to yield an accurate value for G_π from such an over-simplified EPM model

5 Elastic scattering $^3He^3H \to {}^3He^3H$, ref [3] In this process the same mapping method for analytic continuation has been applied as discussed in section 4 for ref [1], and therefore the same criticism applies As the cross section here is dependent on $[G_\pi(q^2)]^4$ the problems are here even more severe Moreover, only points in the neighbourhood of $t = 0$ have been fitted

6 PCAC and analyticity Refs [4,8,9] In ref [8] a quantity $\epsilon(q^2)$ was used, defined by

$$G_P(q^2) = \{[2Mm_\mu G_A(q^2)]/(q^2 + m_\pi^2)\}[1 + \epsilon(q^2)] \tag{5}$$

Eq (4) leads then to

$$G_\pi(-m_\pi^2) = -a_\pi^{-1} G_A(-m_\pi^2)[1 + \epsilon(-m_\pi^2)] \tag{6}$$

From eqs (4) and (5) $\epsilon(q^2)$ has been calculated in ref [8] using $I(q^2)$ from Jarlskog and Ynduraın [6] (see section 3) and a double pole parametrization for $G_A(q^2)$ Then $\epsilon(q^2)$ is found to be nearly constant, $\epsilon(q^2) \approx \epsilon(0) \approx 0\,05$ for $-m_\pi^2 < q^2 < m_\mu^2$, and $G_\pi = -1\,61$ In ref [4], eq (6) has been used together with an analytic extrapolation of $G_A(q^2)$ to the pion pole very different from the double pole extrapolation for $G_A(q^2)$ used in ref [8] to calculate $\epsilon(-m_\pi^2)$ Still the quantity $\epsilon(-m_\pi^2)$ was taken from ref [8] and a low value for G_π found It must be clear now that the procedure of ref [4] is incorrect With the Goldberger Treiman relation [eq (3) for $q^2 = 0$] an analytic continuation for $G_A(q^2)$ is unnecessary and G_π is fixed as soon as $I(0)$ and $G_A(0)$ are known With a different continuation for $G_A(q^2)$, $\epsilon(-m_\pi^2)$ has to be calculated anew, and use of the Jarlskog and Ynduraın value for $I(0)$ and $G_A(0) = 1\,22$, will always result in $G_\pi = -1\,52$

7 Pion photoproduction and low-energy theorems Ref [5] In this reference also a low value, $G_\pi = -1\,02$ was obtained from experimental results on the reaction $\gamma + {}^3H \to \pi^- + {}^3He$ at threshold In the analysis the EPM is used supplied by low-energy theorems

From eq (4) follows $G_\pi(0) = -1\,30$ In a first approximation $G_\pi(q^2)$ is expected to vary with q^2 on a scale connected with the nuclear size, which is dominated by the lowest anomalous thresholds This means that the expansion parameter is not $(m_\pi/M)^2 \approx 0\,0025$ as perhaps naively expected but rather $(m_\pi/1\,8\,m_\pi)^2 \approx 0\,3$ which is large Low-energy theorems determine the amplitude only to first order in k (photon momentum) and q (pion momentum) This means that the difference between $G_\pi(0)$, $G_\pi(-m_\pi^2)$ and e g $G_\pi(+m_\pi^2)$ is already undetermined So the strong variation in $G_\pi(q^2)$ prohibits a precise determination (Cf the situation in ^{6}Li, ref [20])

In this context it may be noted that the equivalence theorem between pseudo-scalar and pseudo-vector couplings holds, up to a small term when the momentum dependence of the form factor is neglected [21] In the catastrophic graph in pseudo-vector theory it is then unclear what the appropriate value for the momentum transfer is when a form factor is inserted Note also that also in this process the IA seems to work rather well [22]

We admit that a similar criticism regarding the use of low-energy theorems applies to the EPM calculations of radiative μ-capture of ref [23] and we think that the impulse approximation calculation there should be taken more serious than the EPM predictions

8 Other determinations Ref [8] As far as is known to us, this are all determinations of G_π leading to low values As we have shown there is no indication that G_π should have such a low value

The high value determined in ref [8] from partial μ-capture $\mu + {}^3He \to {}^3H + \nu_\mu$ has flattered error bars Allowance for a reasonable uncertainty in $G_A(q^2)$ for $q^2 \approx 0\,96\, m_\mu^2$ the momentum transfer at muon capture, leads to no sensible constraint on $G_P(q^2)$ at $q^2 \approx m_\mu^2$ Further the extrapolation to the pion pole is also model dependent

9 Conclusions The value of the concept of a pion–nucleus coupling constant seems to us rather limited In all discussed processes it is the form factor which plays a role, even in pion photoproduction With such a loosely bound system as a nucleus it is very hard to find a way to define and measure G_π, and it is not surprising that different processes and methods yield different outcomes Still we have shown that all published determinations leading to $G_\pi \approx -1$ 0 contain either serious errors or underestimate the uncertainties very much Also, as $G_\pi(0) = -1$ 3 [from eq (3)], there must be in that case a bending in the form factor Only enormous amounts of exchange (speaking in the IA language), or higher threshold diagrams (in dispersion language) can cause such a bending This is rather improbable in view of the good description given by the IA of pion photoproduction [22], radiative pion capture [19], and the charge-exchange reaction $p^3H \to n^3He$ [8]

A high value of $G_\pi \approx -1$ 7 follows in a natural way in the IA from the nuclear form factor and is supported by and consistent with lowest threshold dispersion relations A connection between this result and more model independent ways to measure G_π would, however, be desirable

We thank Professor A D Jackson for helpful discussions This work is part of the research program of the Stichting voor Fundamenteel Onderzoek der Materie (Foundation for Fundamental Research on Matter) and was made possible by financial support from the Nederlandse Organisatie voor Zuiver-Wetenschappelijk Onderzoek (Netherlands Organization for the Advancement of Pure Research)

References

[1] O Dumbrajs, Phys Lett 78B (1978) 24
[2] O Dumbrajs, Lett Nuovo Cimento 29 (1980) 69
[3] O Dumbrajs, Ann Phys (NY) 118 (1979) 249
[4] O Dumbrajs, J Phys G6 (1980) L9
[5] C Leroy and J Pestieau, Phys Lett 82B (1979) 31
[6] C Jarlskog and F J Yndurain, Nuovo Cimento 12A (1972) 801
[7] B Z Kopeliovich, Sov J Nucl Phys 18 (1974) 594
[8] H Primakoff, Nucl Phys A317 (1979) 279
[9] C W Kim and H Primakoff, in Mesons and nuclei, Vol I, eds M Rho and H Wilkinson (North-Holland, Amsterdam, 1979) p 65
[10] L Klieb and H P C Rood, in preparation
[11] J Delorme, in Mesons and nuclei, Vol I, eds M Rho and H Wilkinson (North-Holland, Amsterdam, 1979) p 107
[12] R A Brandenburg, Y E Kim and A Tubis, Phys Rev C12 (1975) 1368
[13] A L L Dieperink, T de Forest, I Sick and R A Brandenburg, Phys Lett 63B (1976) 261
[14] J Delorme, M Ericson, A Figureau and C Thevenet, Ann Phys (NY) 102 (1976) 273
[15] G Plattner and R D Viollier, Nucl Phys A365 (1981) 8
[16] G Bizard et al , Nucl Phys A338 (1980) 451
[17] M P Locher and G Muzitani, J Phys G4 (1978) 287
[18] G Bizard and B Diu, Nuovo Cimento 25A (1975) 467
[19] A C Phillips and F Roig, Nucl Phys A234 (1974) 378
[20] M Moreno, J Pestieau and J Urias, Phys Rev C12 (1975) 514
[21] J L Friar and B F Gibson, Phys Rev C15 (1977) 1779
[22] L Tiator, A K Rej and D Drechsel, Nucl Phys A333 (1980) 343
[23] L Klieb and H P C Rood, Nucl Phys A356 (1981) 483